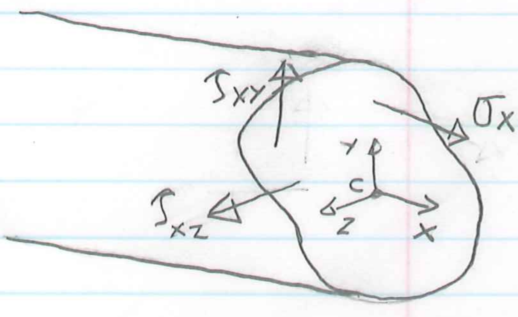
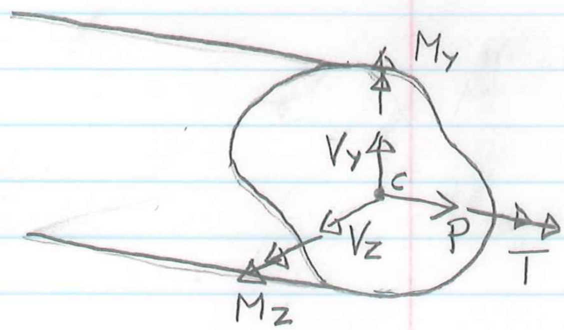
21. Review of relations between stresses and section resultants

x is along the axis of the bar/shaft/beam; y and z are principal, centroidal axes of the cross-section

Stresses on the cross-section are σ_x, τ_{xy} and τ_{xz}.



Section resultants are axial force P and shear forces V_y and V_z (whose lines of action pass through the centroid), twisting coupe T (about x), and bending couples M_y and M_z.



The section resultants are defined in terms of the stresses as follows:

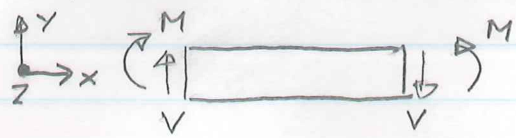
$$P = \int_{c-s} \sigma_x \, dA$$
$$V_y = \int_{c-s} \tau_{xy} \, dA$$
$$V_z = \int_{c-s} \tau_{xz} \, dA$$
$$T = \int_{c-s} (\tau_{xz} \cdot y - \tau_{xy} \cdot z) \, dA$$
$$M_y = \int_{c-s} \sigma_x \cdot z \, dA$$
$$M_z = -\int_{c-s} \sigma_x \cdot y \, dA$$

A structural analysis will determine values of the section resultants as a function of x along a member. The problem then becomes one of finding the stress distributions over the cross-sections. We do this by using St. Venant solutions that are assumed to apply over most of the length of a member. We have already seen that P, M_y and M_z

produce σ_x according to $\sigma_x = \dfrac{P}{A} - \dfrac{M_z \cdot y}{I_z} + \dfrac{M_y \cdot z}{I_y}$.

We have also seen that, for the case of no warping restraint for cross-sections that warp, T produces τ_{xy} and τ_{xz} according to formulas that depend on the shape of the cross-section. In the following sections, we will deal with τ_{xy} and τ_{xz} produced by V_y and V_z.

Actually we will consider only V_y; the discussion for V_z would be similar. Recall from statics that V_y is related to the gradient of M_z, so M_z will exist. Our formulas will also involve I_z. For simplicity in notation, we will use M $(= M_z)$, I $(= I_z)$ and V $(= -V_z)$. The positive sign convention is.



Also, $\dfrac{dM}{dx} = V$.

Example: